

Proper Lucky Labeling of k-Identified Triangular Mesh and k-Identified Sierpiński Gasket Graphs

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Abstract - Let $f: V(G) \rightarrow \mathbb{N}$ be a labeling of the vertices of a graph G by positive integers. Define $S(v) = \sum_{u \in N(v)} f(u)$, as the sum of neighborhood of vertex v , where $N(v)$ denotes the open neighborhood of $v \in V$. A labeling of a graph G is proper lucky labeling if $f(u) \neq f(v)$ and $S(u) \neq S(v)$ for all adjacent vertices u and v in G . The Proper lucky number of G is denoted by $\eta_p(G)$, is the least positive integer k such that G has a proper lucky labeling with $\{1, 2, \dots, k\}$ as the set of labels. In this paper we compute proper lucky number η_p for k-Identified triangular mesh $IT_{(k,n)}$ and for k-Identified Sierpiński Gasket Graph $IS_{(k,n)}$.

Keywords- k -identified triangular mesh, k -identified Sierpiński gasket graph, Proper lucky labeling.

1. INTRODUCTION

Labeling of vertices or edges of graphs is one of the most studied subjects in graph theory. It is assigning the labels to vertices or edges. Labeling is done mostly to distinguish between any two adjacent vertices or edges. But now Labeling of vertices or edges has become very important area of studies due to its applicability in different areas such as computer networking, clustering image sensing, image segmentation etc. Graph labeling was introduced by Rosa in 1967[1]. So many studies have been done in this field. It was later further developed by Graham and Sloane in 1980[2]. We say that a graph has a proper vertex or edge labeling if no two adjacent vertices or edges have the same coloring. The proper Colouring was initiated by Karonski, Luezak and Thomason[3]. If G is any graph whose vertices are arbitrarily labeled and let $S(v)$ denote the sum of labels of all neighbours of vertex $v \in V$, then labeling is lucky if $S(u) \neq S(v)$ if u and v are adjacent vertices of the graph G . The least positive integer k for which a graph G has a lucky labeling from the set $\{1, 2, \dots, k\}$ is called the lucky number of G , denoted by $\eta(G)$. Lucky labeling of graphs were studied in recent times by A. Ahai et al[4] and S. Akbari et al[5]. Proper Lucky labeling is colouring the vertices such that the colouring is proper as well as lucky[6]. The least positive integer k for which a graph G has a lucky labeling from the set $\{1, 2, \dots, k\}$ is called the lucky number of G , denoted by $\eta_p(G)$. The "Sierpiński" type of graphs is naturally available in many different areas of mathematics as well as in several other scientific fields. The Sierpiński Gasket graphs are obtained after a finite number of iterations that in the limit give the Sierpiński Gasket. Such graphs were introduced in 1994 by Scorer, Grundy and Smith [7]. The Sierpiński Gasket graphs play important roles in dynamic systems and probability, as well as in psychology [8]. Sierpiński graphs and Sierpiński Gasket graphs are studied extensively [9, 10, 11]. The definition of triangular mesh, was originally proposed by Razavi and Sarbazi-Azad[12].

In this paper we compute proper lucky number η_p for K-Identified triangular mesh $IT_{(k,n)}$ and for K-Identified Sierpiński gasket Graph $IS_{(k,n)}$.

2. SOME DEFINITIONS AND RESULTS

Definition. 2.1[13]:

Let $f: V(G) \rightarrow \mathbb{N}$ be a labeling of the vertices of a graph G by positive integers. Let $S(v)$ denote the sum of labels over the neighbors of the vertex v in G . If v is an isolated vertex of G we put $S(v) = 0$. A labeling f is lucky if $S(u) \neq S(v)$ for every pair of adjacent vertices u and v . The *lucky number* of a graph G , denoted by $\eta(G)$, is the least positive integer k such that G has a lucky labeling with $\{1, 2, \dots, k\}$ as the set of labels.

Definition. 2.2[6]:

A Lucky labeling is proper lucky labeling if the labeling f is proper as well as lucky, i.e. if $f(u) \neq f(v)$ and $S(u) \neq S(v)$ in a Graph G , for all the adjacent vertices u and v in G . The Proper Lucky number of G is denoted by $\eta_p(G)$, is the least positive integer k such that G has a proper lucky labeling with $\{1, 2, \dots, k\}$ as the set of labels.

Example: Fig 2.1

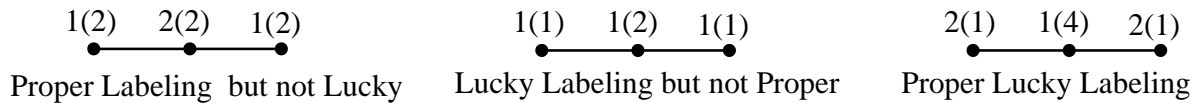


Fig. 2.1

Definition.2.3[14]: A *clique* C in an undirected graph $G = (V, E)$ is a subset of the vertices $C \subseteq V$, such that any two distinct vertices of C are adjacent. A *maximal clique* is a clique which does not exist exclusively within the vertex set of a larger clique. The *clique number* $\omega(G)$ of a graph G is the number of vertices in a maximal clique in G .

Theorem. 2.4[6]: For any connected graph G , $\eta(G) \leq \eta_p(G)$.

Theorem .2.5[6]: For any connected graph G , let ω be its clique number, then $\omega(G) \leq \eta_p(G)$.

Definition. 2.6[15]: A wheel graph W_n is a graph with n vertices ($n \geq 4$), formed by connecting a single vertex to all vertices of an $(n - 1)$ cycles.

Definition.2.7[12]: A Radix- n Triangular mesh network denoted as T_n , consists of a set of vertices $V(T_n) = \{0 \leq x + y < n\}$ where any two vertices (x_1, y_1) and (x_2, y_2) are connected by an edge if $|x_1 - x_2| + |y_1 - y_2| < n - 1$. The number of vertices and edges in a T_n is equal to $\frac{(n+1)(n+2)}{2}$ and $\frac{3n(n-1)}{2}$ respectively. The degree of a vertex in this network can be 2, 4 or 6. The degree of a vertex (x, y) is 2 when $x = y = 0$ or $x = 0$ and $y = n - 1$ or $x = n - 1$ and $y = 0$ and is 4 if $x = 0$ and $1 \leq y \leq n - 2$ or when $y = 0$ and $1 \leq y \leq n - 2$. Otherwise the vertex degree is 6.

Definition.2.8[16]: A k - Identified Triangular Mesh of level n denoted by $IT_{(k,n)}$ is obtained by identifying k copies of Triangular Mesh T_n along their side edges, $k \geq 3$.

Definition 2.9[16]: A k - Identified Sierpiński Gasket of level n denoted by $IS_{(k,n)}$, is obtained by identifying k copies of Sierpiński gasket S_n along their side edges, $k \geq 3$.

Definition 2.10 [16]: The Quotient labeling of $IS_{(k,n)}$ is defined as the graph with center vertex $\langle 0 \dots 0 \rangle$; k special vertices $\langle 0 \dots 0 \rangle, \langle 1 \dots 1 \rangle, \langle 2 \dots 2 \rangle, \dots, \langle k \dots k \rangle$ are called the extreme vertices of $IS_{(k,n)}$, together with vertices of the form $(u_0, u_1 \dots u_r)\{i, j\}$, $1 \leq r \leq n - 2$, $i < j$ where all u_k 's, i and j are from $\{0, 1, \dots, k\}$ and (u_0, u_1, \dots, u_r) is called the prefix of $(u_0, u_1 \dots u_r)\{i, j\}$. Figures 2.2 and 2.3 show a labeled 6-identified Triangular mesh network and a quotient labeled 4-identified Sierpiński gasket graph respectively.

Result.2.11[17]: A wheel graph W_n , $n \geq 4$ has chromatic number is 4 when n is even. A cyclic chromatic number of this graph also 4 and star chromatic number is 4.

Result.2.12[17]: A wheel graph W_n , $n \geq 4$ has chromatic number is 3. When n is odd. A cyclic chromatic number and star chromatic number of these graphs are same as result 2.11.

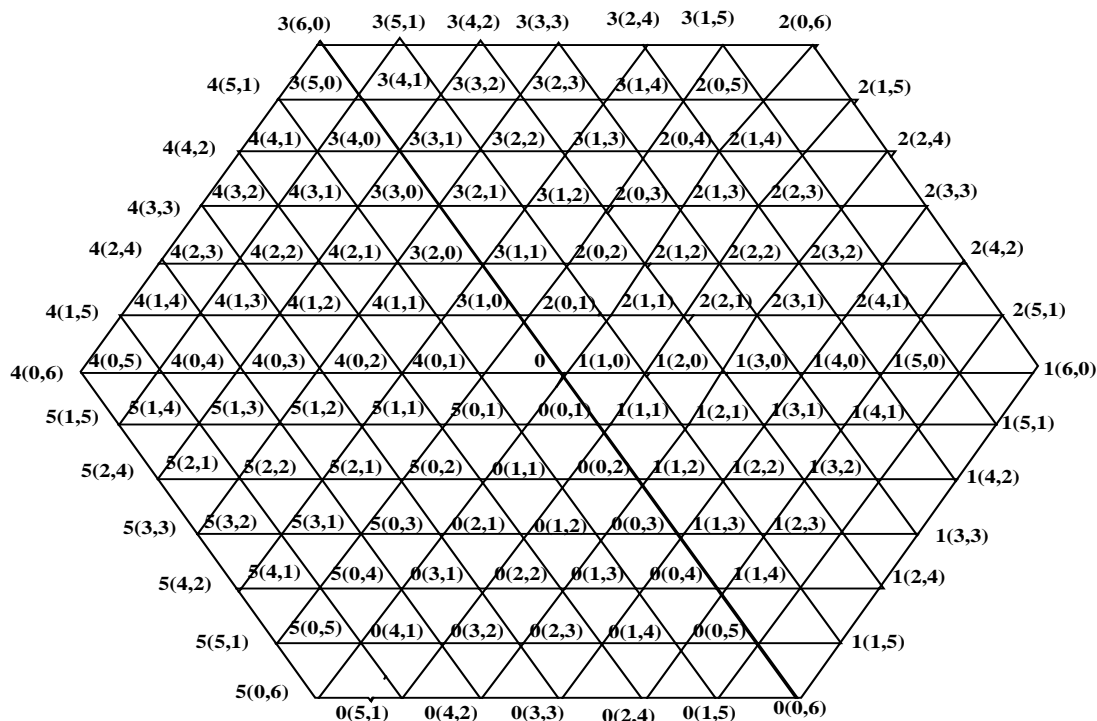


fig. 2.2 $IT_{6,6}$

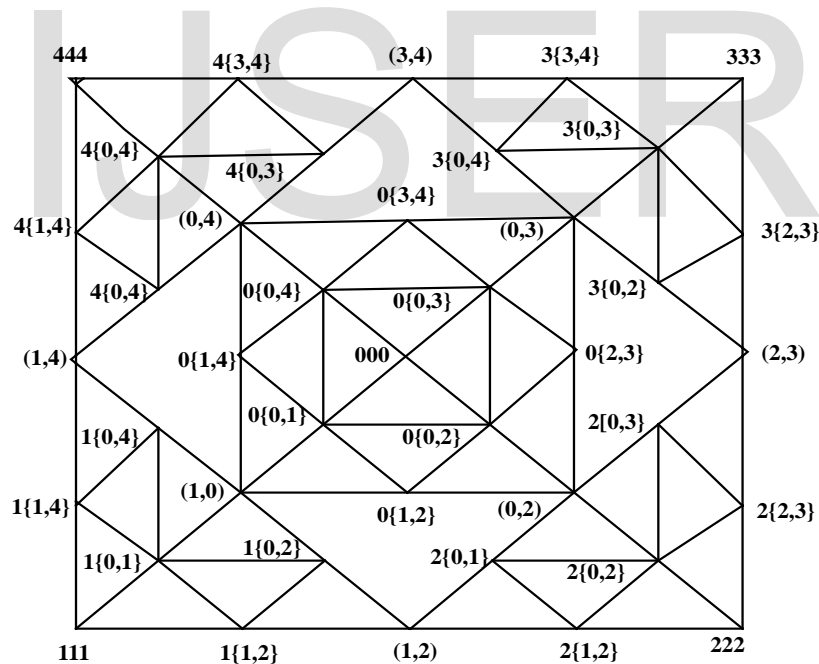


Fig. 2.3 Quotient labeling of $IS_{(4,3)}$

3. PROPER LUCKY LABELING FOR K-IDENTIFIED TRIANGULAR MESH NETWORK

Theorem 3.1: A k –Identified Triangular Mesh $IT_{(k,n)}$ admits proper lucky labeling and

$$\eta_p(IT_{(k,n)}) = 3 \quad \text{when } k \text{ is even} \quad \text{for } n \geq 4.$$

Proof:

Since for a triangular mesh the maximal clique is a triangle and clique number (ω) of any triangle is equal to 3, $\eta_p(IT_{(k,n)}) \geq 3$.

Now to prove the theorem we consider T_2 . It has three copies of triangle of dimension 1. One is at the top and the other two are found to the left and right of the inverted central triangle. Similarly T_3 contains six copies of T_1 , which is just a triangle. So the generalized radix-m triangular mesh T_n will have $\frac{n(n+1)}{2}$ copies of T_1 . We express the triangle T_1 as shown below:

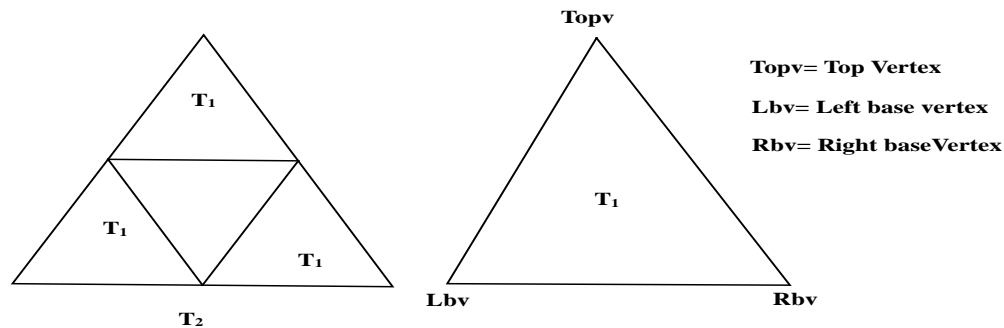


Fig. 3.1 Expression for T_1

While labeling we label Topv first, followed by Lbv and then Rbv. Labeling is done in cyclic order as Topv \rightarrow Lbv \rightarrow Rbv, Lbv \rightarrow Rbv \rightarrow Topv, Rbv \rightarrow Topv \rightarrow Lbv. This order of labeling is always kept along the same direction. We define a map for T_1 since it is the base for k –Identified Triangular Mesh $IT_{(k,n)}$ as $f: V(G) \rightarrow \{1,2,3\}$ defined by $f(Topv) = 1, f(Lbv) = 2, f(Rbv) = 3$. Hence T_1 is labeled in this order; Topv as 1, Lbv as 2 and Rbv as 3. All the copies of T_1 are labeled similarly keeping the order of labeling mentioned above. Label the k –Identified Triangular Mesh $IT_{(k,n)}$ as follows: label 1st copy and all the odd numbered copies of $IT_{(k,n)}$ with 1,2,3 and with its cyclic orders as mentioned above. All the even numbered copies are labeled with 1,3,2 and with its cyclic orders. This labeling is proper because no number (i.e. 1,2,3) is adjacent to itself. In $IT_{(k,n)}$ the degree of vertices are 3,4,6 and the central vertex is of degree k . The corner vertices where the identification takes place is of degree 3, exterior vertices at the circumference of $IT_{(k,n)}$ are of degree 4 and rest of all the vertices are of degree 6. The vertices with degree 3 will have the following neighbourhood sums: if the vertex is labeled as 1, then it will be adjacent to Three vertices of which Two are labeled as 2 and the one vertex is labeled as 3 or Two are labeled as 3 and the one vertex is labeled as 2. So the neighbourhood sum of 1 will be $s(1) = 2(2) + 3 = 7$ or $s(1) = 2(3) + 2 = 8$. If the vertex is labeled as 2 then, it will be adjacent to three vertices of which Two are labeled as 1 and the one vertex is labeled as 3 or Two are labeled as 3 and the one vertex is labeled as 1. The neighbourhood sum of 2 will be $s(2) = 2(1) + 3 = 5$ or $s(2) = 2(3) + 1 = 7$. Similarly if the vertex is labeled as 3, then it will be adjacent to three vertices, of which two are labeled as 1 and the one vertex is labeled as 2 or two are labeled as 2 and the one vertex is labeled as 1. The neighbourhood sum of 3 will be $s(3) = 2(1) + 2 = 4$ or $s(3) = 2(2) + 1 = 5$. The vertices with degree 4 will have the following neighbourhood sums: if the vertex is labeled as 1, then it will be adjacent to four vertices of which two are labeled as 2 and the other two are labeled as 3. So the neighbourhood sum of 1 will be $s(1) = 2(2) + 2(3) = 10$. If the vertex is labeled as 2, then it will be adjacent to four vertices of which two are labeled as 1 and the other two are labeled as 3. So the neighbourhood sum of 2 will be $s(2) = 2(1) + 2(3) = 8$. If the vertex is labeled as 3, then it will be adjacent to four vertices of which two are labeled as 1 and the other two are labeled as 2. So the neighbourhood sum of 3 will be $s(3) = 2(1) + 2(2) = 6$. The vertices with degree 6 will have the following neighbourhood sums: If the vertex is labeled as 1, then it will be adjacent to six vertices of which three vertices are labeled as 2 and the other three vertices are labeled as 3. So the neighbourhood sum of 1 will be $s(1) = 3(2) + 3(3) = 15$. If the vertex is labeled as 2, then it will be adjacent to six vertices of which three vertices are labeled as 1 and the other three vertices are labeled as 3. So the neighbourhood sum of 2 will be $s(2) = 3(1) + 3(3) = 12$. If the vertex is labeled as 3, then it will be adjacent to six vertices of which three vertices are labeled as 1 and the other three vertices are labeled as 2. So the neighbourhood sum of 3 will be $s(3) = 3(1) + 3(2) = 9$. The central vertex

is always labeled as one in this labeling so the neighbourhood sum of the central vertex will be $\frac{k}{2}(5)$. Thus in all cases we observe that no two adjacent vertices have the same neighbourhood sums. (For illustration see figure 3.2)

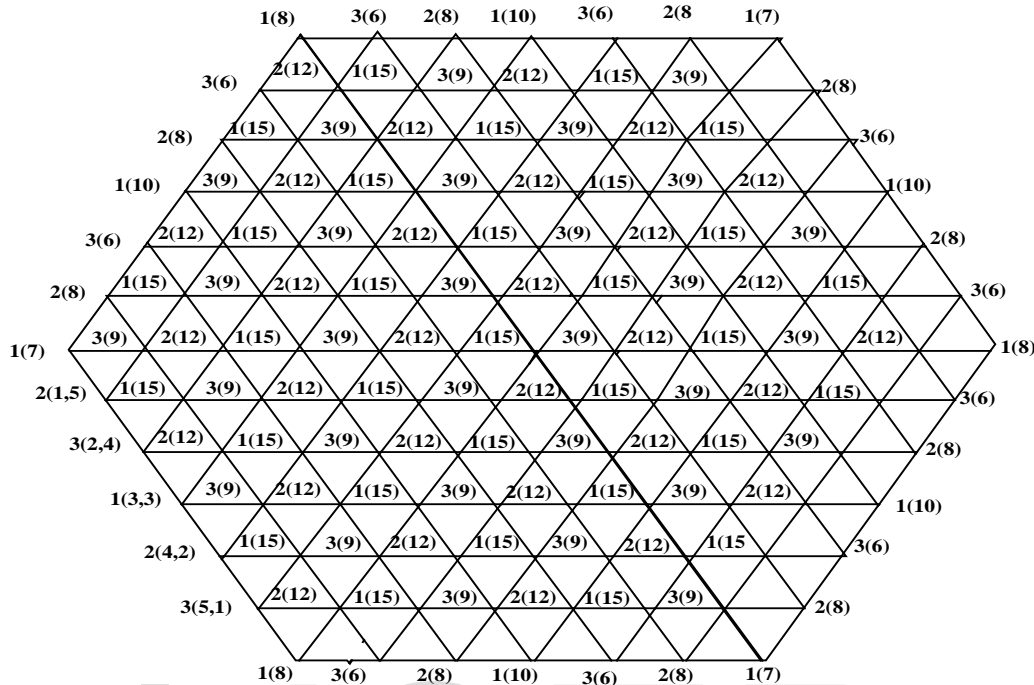


Fig. 3.2 Proper Lucky labeling of $IT_{(6,6)}$

Thus when k is even a k -Identified Triangular Mesh $IT_{(k,n)}$ admits proper lucky labeling and $\eta_p(IT_{(k,n)}) = 3$.

Theorem. 3.2: A k -Identified Triangular Mesh $IT_{(k,n)}$ admits proper lucky labeling and

$$4 \leq \eta_p(IT_{(k,n)}) \leq 5, \quad \text{when } k \text{ is odd,} \quad \text{for } n \geq 4.$$

Proof:

At the centre of $IT_{(k,n)}$ we get an sub-graph which is a wheel graph W_n of even degree. By **Result 2.11** the Chromatic number of a wheel graph of even degree is, $\chi(W_n) = 4$, hence $4 \leq \eta_p(IS_{(k,n)})$.

Label the vertices from 3rd copy till $(k-1)^{th}$ copies of $IT_{(k,n)}$ as above. To label 1st, 2nd and k^{th} copies we have three cases:

Case(i): when $n \bmod 3 \equiv 0$

Label the vertices $k-1(0,0)$ to $k-1(n-2,0)$ of 1st copies as 1,5,3 repeatedly i.e. $k-1(0,0)$ is assigned 1, $k-1(1,0)$ is assigned as 5, and $k-1(2,0)$ is assigned as 3 then again the process is repeated till $k-1(n-2,0)$. $k-1(n-1,0)$ is assigned as 4 and $k-1(n,0)$ is assigned as 1. The vertices from $0(0,1)$ up to $0(n-1,1)$ is labeled as 3,1,2 repeatedly i.e. $0(0,1)$ is assigned 3, $0(1,1)$ is assigned as 1, and $0(2,1)$ is assigned as 2 then again the process is repeated till $0(n-1,1)$. The vertices from $0(0,2)$ till $0(n-2,2)$ is labeled as 4,3,1 as above and the process is repeated ending as vertices $0(n-2,2)$ is assigned as 3. The vertices from $0(0,3)$ up to vertices $0(n-3,3)$ is assigned as 1,2,3 repeatedly and the process is repeated as above. The vertices from $0(0,4)$ up to $0(n-5,4)$ is assigned as 3,1, 2 repeatedly and the process is repeated till $0(n-5,4)$ and vertex $0(n-4,4)$ is assigned as 5. The vertices from $0(0,5)$ up to $0(n-5,5)$ is assigned as 2,3,1 repeatedly and the process is repeated. Then again we repeat the process by assigning vertices $0(0,6)$ till $0(n-6,6)$ with 1,2,3 and repeat the same process. The vertices from $0(0,7)$ till $0(n-7,7)$ is assigned as 3,1,2. Then the cyclic begins for assigning the rest of the vertices. Thus the corner vertex is assigned as 1 i.e. $0(0,n)$ is assigned as 1.

2nd copy of $IT_{(k,n)}$ is labeled as follows: vertex $0(0,0)$ is labeled as 1, vertex $0(1,0)$ is labeled as 3 and vertex $0(2,0)$ is labeled as 4. Then the cyclic order of labeling starts as $0(3,0)$ is labeled as 1, $0(4,0)$ is labeled as 3 and $0(5,0)$ is labeled as 2. The process

continues till $0(n, 0)$ is labeled as 1. The vertices from $1(1, 0)$ till $1(1, n - 1)$ are labeled in cyclic order as 2,1,3 and repeated in cycle ending with vertex $1(1, n - 1)$ labeled as 3. the vertices from $1(2, 0)$ to $1(2, n - 2)$ are labeled in cyclic order as 3,2,1. The vertices from $1(3, 0)$ to $1(3, n - 4)$ is labeled in cyclic order as 1,3,2 ending with $1(3, n - 4)$ labeled as 1. then the process of cycling order of labeling begins again starting with 2,1,3 and process continues till vertex $1(n, 0)$ is labeled as 1.

k^{th} copy of $IT_{(k,n)}$ is labeled as follows: Label the vertices $k - 1(0, 0)$ to $k - 1(n - 2, 0)$ of 1^{st} copies as 1,5,3 repeatedly i.e. $k - 1(0, 0)$ is assigned 1, $k - 1(1, 0)$ is assigned as 5, and $k - 1(2, 0)$ is assigned as 3 then again the process is repeated till $k - 1(n - 2, 0)$. $k - 1(n - 1, 0)$ is assigned as 4 and $k - 1(n, 0)$ is assigned as 1. the vertices from $k - 1(1, 1)$ up to $k - 1(n - 1, 1)$ is labeled in cyclic order as 1,5,4 and repeated ending with vertex $k - 1(n - 1, 1)$ labeled as- 1. The vertices from $k - 1(1, 2)$ up to $k - 1(n - 3, 2)$ are labeled in cyclic order as 2,1,3 and repeated the cycle, and the vertex $k - 1(n - 2, 2)$ is labeled as 4. The vertices from $k - 1(1, 3)$ up to $k - 1(n - 3, 3)$ are labeled in cyclic order as 3,2,1 and repeated the process, ending with vertex $k - 1(n - 3, 3)$ labeled as 5. The vertices from $k - 2(0, 0)$ to $k - 2(0, n)$ are labeled in cyclic order as 1,2,3 and repeated ending with vertex $k - 2(0, n)$ labeled as 1.

In $IT_{(k,n)}$ the degree of vertices are 3,4,6 and the central vertex is of degree k . The neighbourhood sum for 3^{rd} copy till $(k - 1)^{\text{th}}$ copy of $IT_{(k,n)}$ will be the as case 1 mentioned above. The neighbourhood sum for the 1^{st} copy as follows: the vertices which are labeled 1 from $0(0, 1)$ to $0(n - 3, 1)$ will be equal to 20 and vertex $0(n - 2, 1)$ will be equal to 21. The vertices which are labeled as 2 from $0(0, 1)$ to $0(n - 2, 1)$ will have neighbourhood sum as 12 and vertex $0(n - 1, 1)$ will be equal to 9. The vertices which are labeled as 3 from $0(0, 1)$ to $0(n - 3, 1)$ will be equal to 14. The vertices which are labeled as 4 from $0(0, 2)$ to $0(n - 2, 2)$ will be equal to 12. The vertices which are labeled as 3 from $0(0, 2)$ to $0(n - 3, 2)$ will be equal to 11 and the vertex $0(n - 2, 2)$ will be equal to 8. The vertices which are labeled as 1 from $0(0, 2)$ to $0(n - 2, 2)$ will be equal to 17 and the vertex $0(n - 3, 2)$ will be equal to 15. The vertices which are labeled as 3 from $0(0, 4)$ to $0(n - 4, 4)$ will be equal to 9. The vertices which are labeled as 2 from $0(0, 4)$ to $0(n - 4, 4)$ will be equal to 12. The vertices which are labeled as 1 from $0(0, 4)$ to $0(n - 4, 4)$ will be equal to 15. The vertex $0(0, 4)$ will have neighbourhood sum as 8. The vertices which are labeled as 1 from $0(1, 0)$ to $0(1, n - 3, 0)$ will be equal to 22 and the vertex $0(1, n - 2, 0)$ will be equal to 11. the vertices which are labeled as 5 from $0(1, 4, 0)$ to $0(1, n - 2, 0)$ will have neighbourhood sum as 13 and the vertex $0(1, n - 1, 0)$ as 11 and the vertex $0(1, n - 1, 0)$ will have sum as 14. The vertices which are labeled as 3 from $k - 1(1, 0)$ to $0(1, n - 1, 0)$ will be equal to 15. only one vertex $0(1, n - 1, 0)$ is labeled as 4 with neighbourhood sum as 15. The 2^{nd} copy of $IT_{(k,n)}$ has the following neighbourhood sums: The vertices which are labeled as 2 in $1(1, 0)$ to $1(1, n - 1)$ will have neighbourhood sum as 12. The vertices which is labeled as 1 from $1(1, 2)$ to $1(1, n - 1)$ will have neighbourhood sum as 15 and the vertex $1(1, 1)$ will have 17 as its neighbourhood sum. The vertices which are labeled as 3 from $1(1, 3)$ to $1(1, n - 2)$ will have neighbourhood sum as 9 and the vertex $1(1, 2)$ will have sum as 11 and the vertex $1(1, n - 1)$ will have its sum as 6. Rest of the vertices will have neighbourhood sums as in case 1. The $(k - 1)^{\text{th}}$ copy of $IT_{(k,n)}$ will have the following neighbourhood sums: the vertices which are labeled as 2 from $6(0, 2)$ to $6(0, n)$ will have sum as 12 and the vertex $6(0, 1)$ will have its sum as 14. the vertices which are labeled as 3 from $6(0, 1)$ to $6(0, n)$ will have sum as 9. the vertices which are labeled as 1 from $6(0, 1)$ to $6(0, n - 1)$ will have sum as 15 and the vertex $6(0, n)$ will have sum as 7. the vertices which are labeled as 1 from $k - 1(2, 1)$ to $k - 1(n - 1, 1)$ will have sum as 22 and the vertex $k - 1(1, 1)$ will attain its sum as 20. the vertices which are labeled as 5 from $k - 1(1, 1)$ to $k - 1(n - 2, 1)$ will have sum as 12 and the vertex $k - 1(n - 1, 1)$ will have sum as 10. the vertices which are labeled as 4 from $k - 1(1, 1)$ to $k - 1(n - 1, 1)$ will have sum as 16. the vertices which are labeled as 2 from $k - 1(1, 2)$ to $k - 1(n - 2, 2)$ will have sum as 14. the vertices which are labeled as 1 from $k - 1(1, 2)$ to $k - 1(n - 2, 2)$ will have sum as 19. the vertices which are labeled as 3 from $k - 1(1, 2)$ to $k - 1(n - 3, 2)$ will have sum as 11 and the vertex $k - 1(n - 2, 2)$ will have sum as 17. Only one vertex $k - 1(n - 2, 2)$ is labeled as 4 with neighbourhood sum as 14. Rest of the vertices will have neighbourhood sums as case1 except the vertex $k - 1(2, 3)$ labeled as 2 will have sum as 16 and the vertex $k - 1(n - 3, 3)$ with label 5 will have sum as 12 and the $k - 1(n - 2, 4)$ labeled as 3 will have its sum as 10. we notice that labeling is proper and no adjacent vertices have the same neighbourhood sums. (for illustration see figure 3.3).

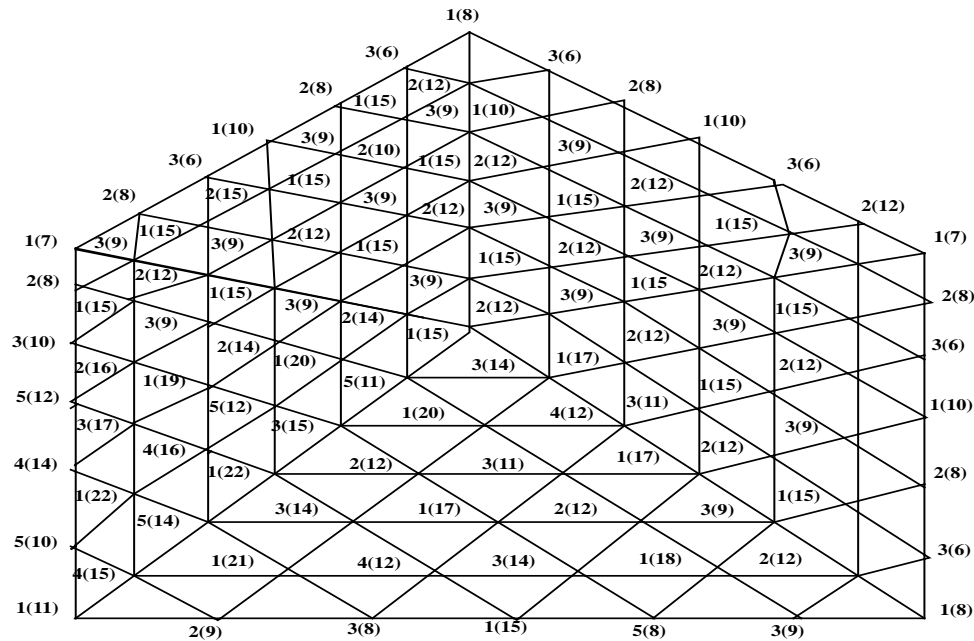


Fig. 3.3 Proper Lucky Labeling of $IT_{(5,6)}$

Case(ii): when $n \bmod 3 \equiv 1$

Labeling is done as in case (i) except the following vertices $k-1(n-1,0)$ is labeled as 1 and has neighbourhood sum as 21, $k-1(n,0)$ is labeled as 5 with sum as 7, $0(n-1,1)$ is labeled as 3 with neighbourhood sum as 9, $0(n-2,2)$ is labeled as 1 with sum as 10, $k-1(3,n-3)$ is labeled as 3 with sum as 12, $k-1(2,n-2)$ is labeled as 1 with sum as 13 and $0(0,n)$ is labeled as 3 with sum as 5. We notice as in subcase(i) no adjacent vertices have the same neighbourhood sums. (for illustration see figure figure 3.4)

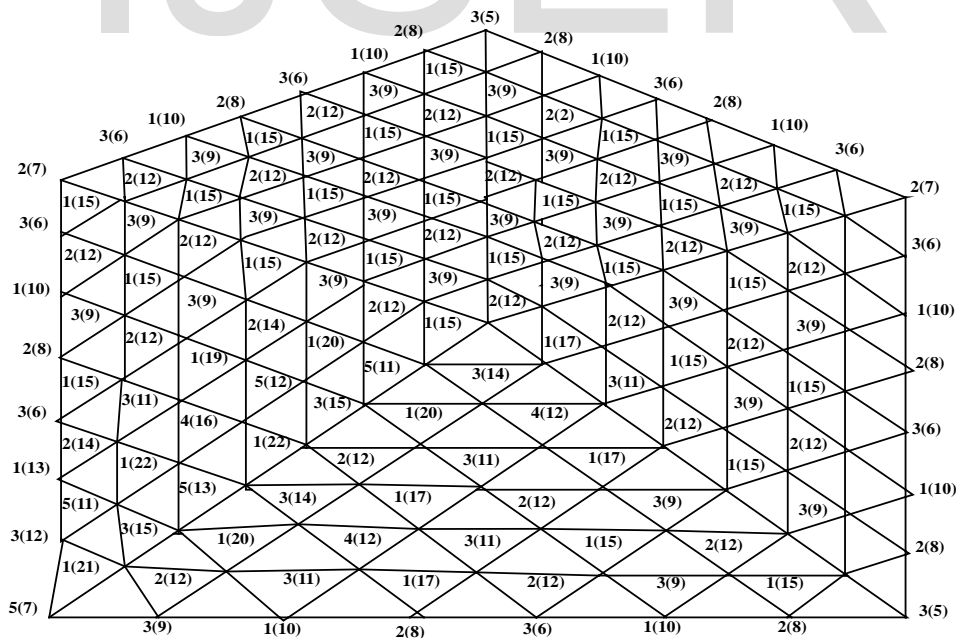


Fig. 3.4 Proper lucky labeling of $IT_{(5,7)}$

Case(iii): when $n \bmod 3 \equiv 2$

Labeling is done as in case (i) except the following vertices $k-1(n-1,0)$ is labeled as 5 and has neighbourhood sum as 14, $k-1(n,0)$ is labeled as 3 with sum as 8, $0(n-1,1)$ is labeled as 1 with neighbourhood sum as 16, $0(n-2,2)$ is labeled as 5 with sum as 8, $0(n-3,3)$ is labeled as 3 with sum as 9, $k-1(2,n-2)$ is labeled as 3 with sum as 9, $k-1(3,n-3)$ is labeled as 2 with sum as 15 and $0(0,n)$ is labeled as 2 with sum as 5. We notice as in subcase(i) no adjacent vertices have the same neighbourhood sums. Rest all the vertices have the same neighbourhood sum as in case1, except the following vertices: $k-1(2,n-3)$ labeled as 4 has sum 17, $k-1(n-2,0)$ labeled as 1 has sum as 21, $0(n-2,1)$ labeled as 3 has sum as 15, $0(n-3,2)$ labeled as 1 has sum as 18. (for illustration see figure 3.5)

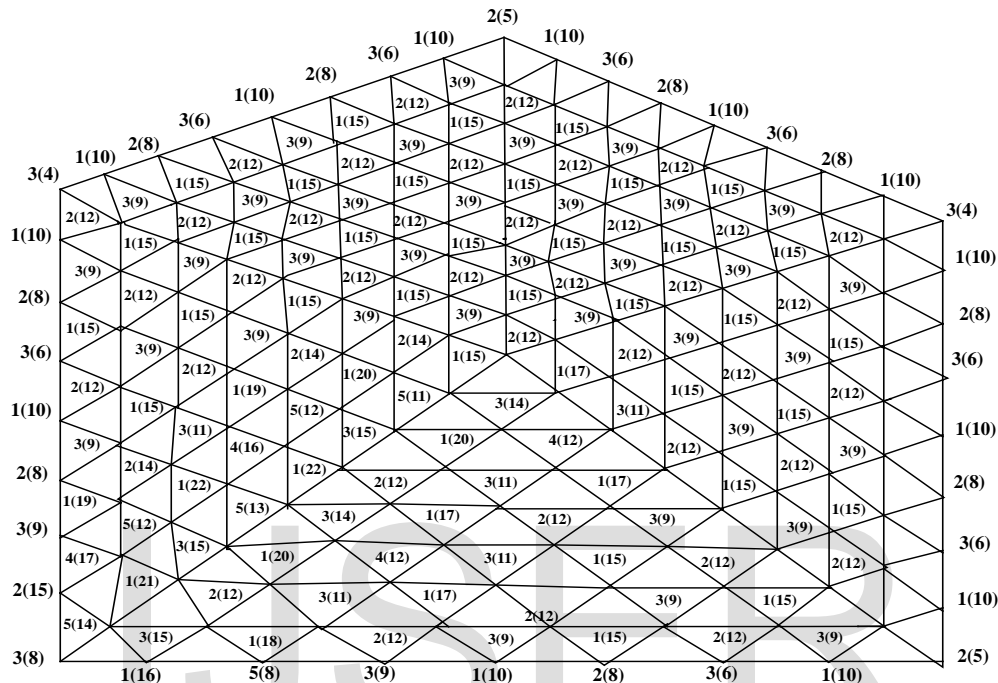


Fig. 3.5 Proper lucky labeling of $IT_{(5,8)}$

Hence a k -Identified Triangular Mesh $IT_{(k,n)}$ admits proper lucky labeling and

$$\eta_p(IT_{(k,n)}) \leq 5 \text{ when } k \text{ is odd, for } n \geq 4.$$

4. PROPER LUCKY LABELING FOR K-IDENTIFIED SIERPIŃSKI GASKET NETWORKS

4.1 Theorem: A k -Identified Sierpiński gasket networks $IS_{(k,n)}$ admits proper lucky labeling and

$$\eta_p(IS_{(k,n)}) = 3 \text{ when } k \text{ is even} \quad \text{for } n \geq 4.$$

Proof:

By similar argument of theorem 3.1 we have $\eta_p(IS_{(k,n)}) \geq 3$. To prove the theorem we consider S_3 . It has three copies of triangle of dimension 2. One is at the top and the other two are found to the left and right of the inverted central triangle. Similarly S_2 contains three copies of S_1 , one located at the top and other two are found to the left and right of the centered inverted triangle, where S_1 is just a triangle. So the generalized Sierpiński network S_n will have three copies of $S_{(n-1)}$. We express the triangle S_1 as shown in the diagram below:

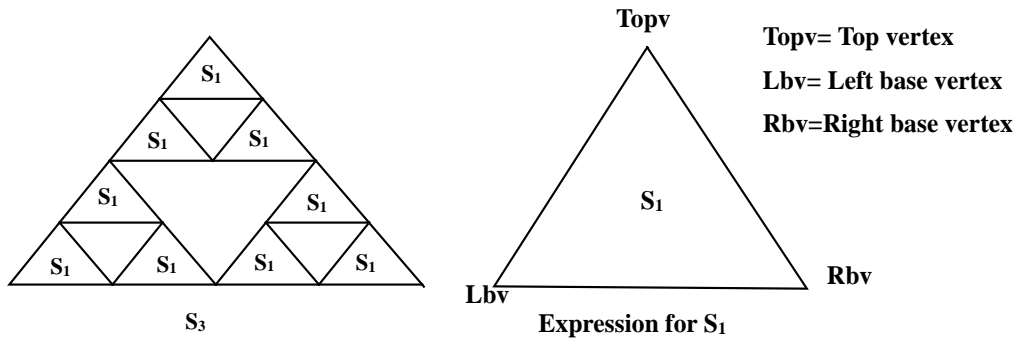


Fig. 4.1 S_3 and Expression for S_1

While labeling we label Topv first, followed by Lbv and then Rbv. Labeling is done in cyclic order as Topv \rightarrow Lbv \rightarrow Rbv, Lbv \rightarrow Rbv \rightarrow Topv, Rbv \rightarrow Topv \rightarrow Lbv. This order of labeling is always kept along the same direction. We define a map for S_1 since it is the base for K-identified Sierpiński network $IS_{(k,n)}$ as:

$f : V(G) \rightarrow \{1,2,3\}$ defined by $f(Topv) = 1, f(Lbv) = 2, f(Rbv) = 3$. Hence S_1 is labeled in this order; Topv as 1, Lbv as 2 and Rbv as 3. All the copies of S_1 are labeled similarly keeping the order of labeling mentioned above. Now k-identified Sierpiński network $IS_{(k,n)}$ is actually k copy of S_n . Hence labeling for the copy of $IS_{(k,n)}$ is done similarly as for S_n .

The degree of vertices in k-identified Sierpiński network $IS_{(k,n)}$ are 3, 4, 6 and K. The extreme vertices $\langle 1 \dots 1 \rangle, \langle 2 \dots 2 \rangle, \dots, \langle k \dots k \rangle$ are of degree 3, all the vertices are of degree 4 except the vertex at the join of two S_n 's such as $22 \dots \{2,3\}, 2 \dots \{2,3\}, 23 \dots \{2,3\}, \dots, (2,3), 32 \dots \{2,3\}, \dots, 3 \dots \{2,3\}, \dots, 33 \dots \{2,3\}, \dots$, etc. are of degree 6. The vertices with degree 3 will have the following neighbourhood sums: if the vertex is labeled as 1, then it will be adjacent to three vertices of which two are labeled as 2 and the other is labeled as 3 or two vertices labeled as 3 and other is labeled as 3. So the neighbourhood sum of 1 will be $s(1) = 2(2) + 3 = 7$ or $s(1) = 2(3) + 2 = 8$. if the vertex is labeled as 2, then it will be adjacent to three vertices of which two are labeled as 1 and the other is labeled as 3 or two vertices labeled as 3 and other is labeled as 1. So the neighbourhood sum of 2 will be $s(2) = 2(1) + 3 = 5$ or $s(2) = 2(3) + 1 = 7$. if the vertex is labeled as 3, then it will be adjacent to three vertices of which two are labeled as 1 and the other is labeled as 2 or two vertices labeled as 2 and other is labeled as 1. So the neighbourhood sum of 3 will be $s(3) = 2(1) + 2 = 4$ or $s(3) = 2(2) + 1 = 5$. The vertices with degree 4 will have the following neighbourhood sums: if the vertex is labeled as 1, then it will be adjacent to four vertices of which two are labeled as 2 and the other two are labeled as 3. So the neighbourhood sum of 1 will be $s(1) = 2(2) + 2(3) = 10$. If the vertex is labeled as 2, then it will be adjacent to four vertices of which two are labeled as 1 and the other two are labeled as 3. So the neighbourhood sum of 2 will be $s(2) = 2(1) + 2(3) = 8$. If the vertex is labeled as 3, then it will be adjacent to four vertices of which two are labeled as 1 and the other two are labeled as 2. So the neighbourhood sum of 3 will be $s(3) = 2(1) + 2(2) = 6$. The vertices with degree 6 will have the following neighbourhood sums: If the vertex is labeled as 1, then it will be adjacent to six vertices of which three vertices are labeled as 2 and the other three vertices are labeled as 3. So the neighbourhood sum of 1 will be $s(1) = 3(2) + 3(3) = 15$. If the vertex is labeled as 2, then it will be adjacent to six vertices of which three vertices are labeled as 1 and the other three vertices are labeled as 3. So the neighbourhood sum of 2 will be $s(2) = 3(1) + 3(3) = 12$. If the vertex is labeled as 3, then it will be adjacent to six vertices of which three vertices are labeled as 1 and the other three vertices are labeled as 2. So the neighbourhood sum of 3 will be $s(3) = 3(1) + 3(2) = 9$. And the central vertex will have neighbourhood sum as $\frac{k}{2}(5)$. Thus in all cases we observe that no two adjacent vertices have the same neighbourhood sums. (for illustration see figure 4.2)

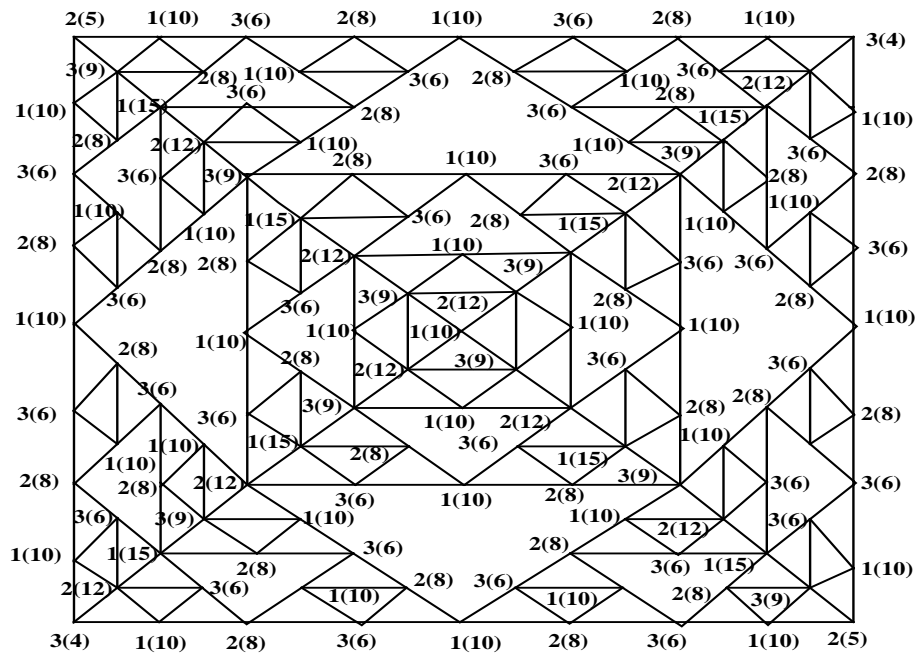


Fig. 4.2 Lucky labeling of $IS_{4,4}$

Thus when k is even a k -Identified Sierpiński Gasket graph $IS_{(k,n)}$ admits proper lucky labeling and $\eta_p(IS_{(k,n)}) = 3$.

Theorem 4.2: A k -Identified Sierpiński gasket networks $IS_{(k,n)}$ admits proper lucky labeling and

$$4 \leq \eta_p(IS_{(k,n)}) \leq 5 \text{ when } k \text{ is odd} \quad \text{for } n \geq 4.$$

Proof: By similar argument as Theorem 3.2, we have $4 \leq \eta_p(IS_{(k,n)})$.

Label the vertices from 3rd copy till $(k-1)^{th}$ copies of $IS_{(k,n)}$ as in Theorem 4.1. To label 1st, 2nd and k^{th} copies we have two cases:

Case(i): when k is even

Label 1st, 2nd and k^{th} copies of $IS_{(k,n)}$ vertices $\{0 \dots 0\}, 0 \dots \{0,2\}, \dots, \{0,2\}, \dots, 2 \dots \{0,2\}$ as 1,5,3 repeatedly i.e. $\{0 \dots 0\}$ is assigned 1, $0 \dots \{0,2\}$ is assigned as 5, and $0 \dots 2\{0,2\}$ is assigned as 3 then again the process is repeated in cyclic way. The vertex $0 \dots 2\{0,1\}$ is assigned 1 and the vertices from $0 \dots 2\{1,2\}, \dots, 2 \dots 0\{0,1\}, \dots, 22 \dots \{0,1\}$ is labeled as 5,4,4,5,4,2 repeatedly i.e. $0 \dots \{0,1\}$ is assigned as 5, and $0 \dots 2\{1,2\}$ is assigned as 4 and so on and the process is repeated and $0 \dots \{1,2\}$ is assigned as 1. The vertex $2 \dots \{1,2\}$ is assigned 2. 2nd copy of $IS_{(k,n)}$ is labeled as case 1 except the vertices next to $\{0 \dots 0\}$, $0 \dots \{0,3\}$ is labeled as 4 and next to it $00\{0,3\}$ is labeled as 5 and $22 \dots \{2,3\}$ i.e. third vertex to the right of $\{2 \dots 2\}$ in the base level is labeled as 5.

In $IS_{(k,n)}$ the degree of vertices are 3, 4, 6 and the central vertex is of degree k . The neighbourhood sum for 3rd copy till $(k-1)^{th}$ copy of $IS_{(k,n)}$ will be same as in Theorem 4.1.

The neighbourhood sum for the 1st, 2nd and k^{th} copies of $IS_{(k,n)}$ we notice that no adjacent vertices have the same neighbourhood sums due to our labeling. (for illustration see figure 4.3)



Labeling is done similar to Case(i) except the vertices $0 \dots \{0,3\}$ is labeled as 5 and $22 \dots \{2,3\}$ i.e. third vertex to the right of $\langle 2 \dots 2 \rangle$ in the base level is labeled as 5. In this case also no adjacent vertices have the same neighbourhood sums. (for illustration see figure 4.4)

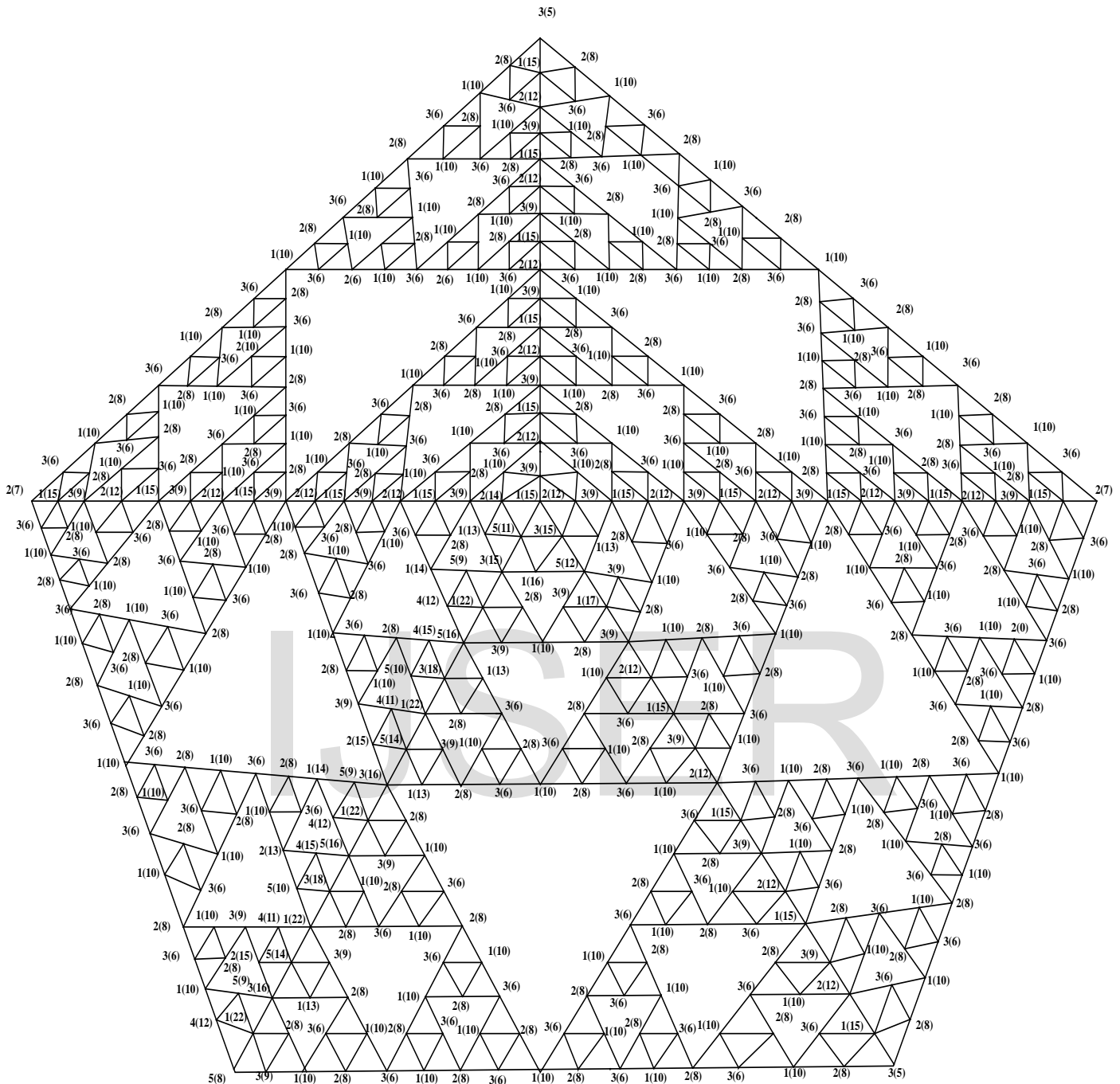


Fig. 4.4 Proper Lucky labeling of $IS_{(5,5)}$

Hence, a k –Identified Sierpiński gasket networks $IS_{(k,n)}$ admits proper lucky labeling and

$$n_p(IS_{(k,n)}) \leq 5 \text{ when } k \text{ is odd,} \quad \text{for } n \geq 4.$$

5. CONCLUSION:

We computed proper lucky number for k -identified Triangular Mesh Networks and k -identified Sierpiński gasket networks for even k and found the upper bound for both for odd k .

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